Mathematics Pre-Course Work

1. Fractions

Examples: Calculate 1) $1\frac{2}{3}+1\frac{1}{2}$ 2) $\frac{3}{8}\times\frac{4}{9}$ and 3) $2\frac{1}{4}\div\frac{3}{5}$

Solution 1) $1\frac{2}{3}+1\frac{1}{2}$

$=\frac{5}{3}+\frac{3}{2}$

$=\frac{10}{6}+\frac{9}{6}$

$=\frac{19}{6}$

$=3\frac{1}{6}$

Solution 2) $\frac{3}{8}\times\frac{4}{9}$

$=\frac{\cancel{3}^{1}}{\cancel{8}^{2}}\times\frac{\cancel{4}^{1}}{\cancel{9}_{3}}$

$=\frac{1}{2}\times\frac{1}{3}$

$=\frac{1}{6}$

Solution 3) $2\frac{1}{4}\div\frac{3}{5}$

$=\frac{9}{4}\div\frac{3}{5}$

$=\frac{\cancel{9}^{3}}{4}\times\frac{5}{\cancel{3}^{1}}$

$=\frac{3}{4}\times\frac{5}{1}$

$=\frac{15}{4}$

Your Turn:

a) $\frac{3}{5}-\frac{1}{5}=$

b) $\frac{3}{7}-\frac{4}{7}=$

c) $\frac{2}{5}+\frac{3}{10}=$

d) $2\frac{1}{5}+\frac{3}{10}=$

e) $\frac{3}{8}-\frac{1}{6}=$

f) $1\frac{9}{10}-\frac{1}{3}=$

g) $\frac{2}{3}\times 3\frac{4}{5}=$

h) $2\frac{4}{5}\times 4\frac{5}{6}=$

i) $1\div\frac{1}{5}=$

j) $\frac{2}{3}\div 1\frac{1}{9}=$

2. Laws of Indices

Examples: Simplify, writing as a single power. 1) $4^2 \times 4^5$ 2) $4^9 \div 4^3$ 3) $(5^2)^5$

Solution 1) $4^2 \times 4^5$
$= 4^{2+5}$
$= 4^7$

Solution 2) $4^9 \div 4^3$
$= 4^{9-3}$
$= 4^6$

Solution 3) $(5^2)^5$
$= 5^{2\times 5}$
$= 5^{10}$

Your Turn:

a) $2^2 \times 2^5 =$

b) $3^4 \div 3^3 =$

c) $4^7 \div 4^3 \times 4^2 =$

d) $(x^5)^3 =$

e) $(a^4)^3 \div (a^2)^3 =$

f) $\frac{c^3\times c^2}{c^7} =$

Examples: Calculate the value without a calculator. 1) 5^{-3} 2) $8^{1/3}$

Solution 1) 5^{-3}

$=\frac{1}{5^3}$

$=\frac{1}{125}$

Solution 2) $8^{1/3}$

$=\sqrt[3]{8}$

$= 2$

Your Turn:

g) $3^{-2} =$

h) $7^0 =$

i) $2 \times 3^3 =$

j) $2^2 \times 3^2 =$

k) $64^{\frac{1}{2}} =$

l) $27^{\frac{1}{3}}$

3. Surds

Examples: Simplify, writing as a single surd where possible 1) $\sqrt{3} + 2\sqrt{3}$ 2) $\sqrt{2} \times \sqrt{5}$ 3) $\sqrt{90}$

Solution 1) $\sqrt{3} + 2\sqrt{3}$
$= 3\sqrt{3}$

Solution 2) $\sqrt{2} \times \sqrt{5}$
$= \sqrt{2 \times 5}$
$= \sqrt{10}$

Solution 3) $\sqrt{90}$
$= \sqrt{9 \times 10}$
$= \sqrt{9} \times \sqrt{10}$
$= 3\sqrt{10}$

Your Turn:

a) $3\sqrt{7} + 2\sqrt{7} =$

b) $4\sqrt{2} - 3\sqrt{2} =$

c) $3\sqrt{7} \times \sqrt{7} =$

d) $3\sqrt{6} \times \sqrt{6} =$

e) $\sqrt{18} =$

f) $\sqrt{32} =$

g) $\sqrt{54} =$

h) $\sqrt{12} =$

4. Expanding and Simplifying Expressions

Examples: Expand and simplify 1) $4(x + 2) - 3(x - 1)$ and 2) $(x + 4)(x - 6)$

Solution 1) $4(x + 2) - 3(x - 1)$
$= 4x + 8 - 3x + 3$
$= 4x - 3x + 8 + 3$
$= x + 11$

Solution 2) $(x + 4)(x - 6)$
$= x^2 - 24 + 4x - 6x$
$= x^2 - 2x - 24$

Your Turn:

a) $6b^2 + 5b - 1 + 3b + 4$

b) $5(x - 3)$

c) $-2(3x + 1)$

d) $3(4x + 2) + 5(2x - 1)$

e) $5(2x - 4) - 2(3x - 7)$

f) $(x - 1)(x + 1)$

g) $(3a + 2)(a - 1)$

h) $(2b - 3)(3b - 2)$

i) $4(xy)$

j) $(3x)^2$

k) $\frac{3x+6y}{3}$

l) $\frac{4}{2x+4}$

m) $\frac{2}{5x-2}$

5. Factorising

Examples: 1) $9xy + 15x$ 2) $x^2 + 3x + 2$ 3) $x^2 - 9$

Solution 1) $9xy + 15x$ write the highest common factor, HCF, outside the brackets
$3x(\quad\quad)$ divide both parts of the expression by the HCF
$3x(3y + 5)$ check your answer by multiplying through the brackets.

Solution 2) $x^2 + 3x + 2$ — Set out double brackets, writing an x in each one

$(x \quad)(x \quad)$ — Think of two factors of 2 that will add to 3.

$(x + 2)(x + 1)$

Solution 3) $x^2 - 9$ — 9 and x^2 are both square numbers; this is a DOTS question!

$(x + 3)(x - 3)$ — add and subtract the square roots in the brackets.

Your Turn:

a) $4x + 8 =$

b) $2ab + ad$

c) $8x^2 - 10x$

d) $8ab^2 - 4a^2b$

e) $x^2 + 4x + 3$

f) $x^2 + 8x + 15$

g) $x^2 + 12x - 28$

h) $x^2 - 17x + 30$

i) $x^2 - 36$

j) $2x^2 + 7x + 5$

6. Solving Linear Equations

Examples:

Solve the following equations 1) $5x+4=11$ and 2) $7(x-2)=7$

Solution 1)

$$5x+4=11$$
$$5x=11-4$$
$$5x=7$$
$$x=7\div 5=\tfrac{7}{5}$$

Solution 2)

$$7(x-2)=7$$
$$7x-14=7$$
$$7x=7+14=21$$
$$x=21\div 7=3$$

Your Turn:

a) $5x + 7 = 32$

b) $2(2x - 7) = 7$

c) $4x - 5 = 2x + 7$

d) $3p + 2 = 5 - p$

e) $2 - 3(2x - 5) = 7 - x$

f) $\frac{3x-13}{7} + \frac{11-4x}{3} = 0$

g) $\frac{6}{x} + \frac{3}{2x} = \frac{5}{2}$

7. Formulae

Examples: Substitute into the following formulae to determine the missing value 1) If $x = ab - c$, find x when $a = 4$, $b = \tfrac{1}{2}$ and $c = -5$

$$x = ab - c$$
$$= 4\times\tfrac{1}{2} - (-5) \qquad 4\times\tfrac{1}{2} = 2 \text{ and } -(-5) \text{ is the same as } +5$$
$$= 2 + 5$$
$$= 7$$

Your Turn:

a) $x = ab + c$ Find x when $a = \frac{2}{3}$, $b = 9$ and $c = -3$

b) $x = 2a^2$ Find x when $a = \frac{3}{4}$

c) $A = 4\pi r^2$ Find r when A = 616

d) a = b – ½c Find c when a = 6 and b = 10

e) v = u + at Find a when v = 21.5, u = 4 and t = 7

Examples: Make x the subject of each of these formulae;

1) $a = x - ab$ 2) $xy = w$ and 3) $f = d(x+e)$

Solution 1)

$a = x - ab$ Treat ab as a single item; add ab to each side

$a + ab = x$ Swap each side to give $x =$

$x = a + ab$

Solution 2)

$xy = w$ Remember that $xy = x \times y$, you need to divide by y

$$x = \frac{w}{y}$$

Solution 3)

$f = d(x+e)$ Firstly multiply out the brackets

$f = dx + de$ Treating de as a single item; subtract de from each side

$f - de = dx$ Divide by d

$\frac{f-de}{d} = x$ Swap each side to give $x =$

$$x = \frac{f-de}{d}$$

Your Turn:

f) 3x = b

g) $\frac{x}{5} = d$

h) f = 4 – x

i) 2(3x – 1) = 5y

j) ax = bx + c

k) mx = u – 2x

l) $\sqrt{x-2} = y$

8. Solving Quadratic Equations

Examples:

Solve the following quadratic equations; 1) $x^2 - 8x + 12 = 0$ and 2) $y^2 + 13y + 40 = 0$.

Solution 1)

In the quadratic equation $x^2 - 8x + 12 = 0$, the expression can be factorised. So (x – 6)(x – 2) = 0

We set each factor pair equal to zero to get our two solutions.

x – 6 = 0 and x – 2 = 0

x = 6 or 2

Solution 2)

In the equation $y^2+13y+40=0$, we have $a=1$, $b=13$ and $c=40$. So

$$y=\frac{-13\pm\sqrt{13^2-4\times1\times40}}{2\times1}=\frac{-13\pm\sqrt{169-160}}{2}=\frac{-13\pm\sqrt{9}}{2}=\frac{-13\pm3}{2}=\frac{-13+3}{2}\text{ or }\frac{-13-3}{2}=-5\text{ or }-8$$

Your Turn:

a) $n^2 + 5n + 4 = 0$

b) $t^2 - 4t - 12 = 0$

c) $x^2 - 81 = 0$

d) $x^2 - 2x - 6 = 0$

e) $x^2 - 6x - 8 = 0$

f) $3x^2 + 10x - 7 = 0$

9. Simultaneous Linear Equations

Examples:

Solve the following pairs of simultaneous equations

1) $$\begin{aligned}7x+2y&=32\\ x+y&=1\end{aligned}$$

2) $$\begin{aligned}5x+2y&=26\\ 4x-3y&=7\end{aligned}$$

Solution 1)

Double the second equation to give

$$\begin{aligned}7x+2y&=32\\ 2x+2y&=2\end{aligned}$$

Subtract the new second equation from the new first, and solve the resulting equation to find x

$$\begin{aligned}5x&=30\\ x&=6\end{aligned}$$

Substitute into either of the original equations to find y

$$\begin{aligned}x+y&=1\\ \Rightarrow\quad 6+y&=1\\ y&=-5\end{aligned}$$

Solution 2)

Multiply the first equation by 3 and the second equation by 2 to give

$$\begin{aligned}15x+6y&=78\\ 8x-6y&=14\end{aligned}$$

Add the two equations and solve

$$\begin{aligned}23x&=92\\ x&=4\end{aligned}$$

Substitute into either of the original equations to find y

$$\begin{aligned}5x+2y&=26\\ \Rightarrow 20+2y&=26\\ 2y&=6\\ y&=3\end{aligned}$$

Your Turn:

a) 5x – 3y = 23
2x + 3y = 26

b) y = 2x + 1
3y + 10x = 7

c) 5x + 2y = 11
3x + 7y = -5

d) x + 2y = 4
2x + y = 5

e) 3x – 6y = 33
x – 3y = 16

11. Completing the Square

https://www.mathsisfun.com/algebra/completing-square.html

Express in the form $(x+a)^2+b$

a x^2+2x+4	**b** x^2-2x+4	**c** x^2-4x+1	**d** x^2+6x
e x^2+4x+8	**f** x^2-8x-5	**g** $x^2+12x+30$	**h** $x^2-10x+25$
i x^2+6x-9	**j** $18-4x+x^2$	**k** x^2+3x+3	**l** x^2+x-1

Quadratic Equation can have a coefficient of a in front of x^2:

$$ax^2 + bx + c = 0$$

But that is easy to deal with ... just divide the whole equation by "a" first, then carry on:

$$x^2 + (b/a)x + c/a = 0$$

Express in the form $a(x+b)^2+c$

a $2x^2+4x+3$	**b** $2x^2-8x-7$	**c** $3-6x+3x^2$	**d** $4x^2+24x+11$
e $-x^2-2x-5$	**f** $1+10x-x^2$	**g** $2x^2+2x-1$	**h** $3x^2-9x+5$
i $3x^2-24x+48$	**j** $3x^2-15x$	**k** $70+40x+5x^2$	**l** $2x^2+5x+2$
m $4x^2+6x-7$	**n** $-2x^2+4x-1$	**o** $4-2x-3x^2$	**p** $\frac{1}{3}x^2+\frac{1}{2}x-\frac{1}{4}$

Expand out your answers to check your work.
Why do we complete the square?
Find the turning point of each of your answers above.

12. Inequalities

Linear inequalities are solved in the same way as linear equations. But you must keep one thing in mind: multiplying or dividing by a negative number causes the inequality to change direction. Often it is worth finding a way around dividing by the negative number in the first place to avoid mistakes.

Q1) $8x+7<47$	Q5) $4x+1>3x+3$
Q2) $4x-8<20$	Q6) $2x+54<10x+6$
Q3) $4x+3>43$	Q7) $7x+1<8x-3$
Q4) $4x-5\geq -17$	Q8) $6x+15\geq 8x-5$

14. Harder simultaneous equations

More difficult simultaneous equations involve quadratics and need to be solved using **substitution**. This type of question gives 4 answers, rather than 2 in normal simultaneous equations.

Examples:

Solve the following pairs of simultaneous equations

$$x^2 + y^2 = 100$$
$$x - y = 2$$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$$x = y + 2$$

Step 2- substitute into the quadratic and simplify

$(y + 2)^2 + y^2 = 100$
$y^2 + 4y + 4 + y^2 = 100$
$2y^2 + 4y - 96 = 0$
$y^2 + 2y - 48 = 0$

Step 3- Solve by factorising or using the formula

$(y + 8)(y - 6) = 0$
$y = -8$ or $y = 6$

Step 4- Substitute your two values into the linear equation to find the other solutions

$x = y + 2$
When $y = -8$, $x = -8 + 2 =$ **-6**
When $y = 6$, $x = 6 + 2 =$ **8**

$$y = x^2 - 3x + 4$$
$$y - x = 1$$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$$y = 1 + x$$

Step 2- substitute into the quadratic and simplify

$1 + x = x^2 - 3x + 4$
$x^2 - 4x + 3 = 0$

Step 3- Solve by factorising or using the formula

$(x - 1)(x - 3) = 0$
$x = 1$ or $x = 3$

Step 4- Substitute your two values into the linear equation to find the other solutions

$y = 1 + x$
When $x = 1$, $y = 1 + 1 =$ **2**
When $x = 3$, $y = 1 + 3 =$ **4**

Your Turn:

1. $y = x^2 + 7x - 2$
   $y = 2x - 8$

2. $x^2 + y^2 = 8$
   $y = x + 4$

3. $y = x^2$
   $y = x + 2$

4. $x^2 + y^2 = 5$
   $x - 2y = 5$

5. $x^2 + y^2 = 36$
   $x = 2y + 6$

6. $x^2 + y^2 = 25$
   $y = 2x + 5$

7. $x^2 + y^2 = 9$
   $x + y = 2$