Mathematics Pre-Course Work

1. Fractions

Examples: Calculate 1) $1\frac{2}{3}+1\frac{1}{2}$ 2) $\frac{3}{8}\times\frac{4}{9}$ and   3) $2\frac{1}{4}\div\frac{3}{5}$

Solution 1) $1\frac{2}{3}+1\frac{1}{2}$

$=\frac{5}{3}+\frac{3}{2}$

$=\frac{10}{6}+\frac{9}{6}$

$=\frac{19}{6}$

$=3\frac{1}{6}$

Solution 2) $\frac{3}{8}\times\frac{4}{9}$

$=\frac{\not{3}^1}{\not{8}^2}\times\frac{\not{4}^1}{\not{9}_3}$

$=\frac{1}{2}\times\frac{1}{3}$

$=\frac{1}{6}$

Solution 3) $2\frac{1}{4}\div\frac{3}{5}$

$=\frac{9}{4}\div\frac{3}{5}$

$=\frac{\not{9}^3}{4}\times\frac{5}{\not{3}^1}$

$=\frac{3}{4}\times\frac{5}{1}$

$=\frac{15}{4}$

Your Turn:

a) $\frac{3}{5}-\frac{1}{5}=$

b) $\frac{3}{7}-\frac{4}{7}=$

c) $\frac{2}{5}+\frac{3}{10}=$

d) $2\frac{1}{5}+\frac{3}{10}=$

e) $\frac{3}{8}-\frac{1}{6}=$

f) $1\frac{9}{10}-\frac{1}{3}=$

g) $\frac{2}{3}\times3\frac{4}{5}=$

h) $2\frac{4}{5}\times4\frac{5}{6}=$

i) $1\div\frac{1}{5}=$

j) $\frac{2}{3}\div1\frac{1}{9}=$

2. Laws of Indices

Examples: Simplify, writing as a single power. 1) $4^2\times4^5$ 2) $4^9\div4^3$ 3) $(5^2)^5$

Solution 1) $4^2\times4^5$
$=4^{2+5}$
$=4^7$

Solution 2) $4^9\div4^3$
$=4^{9-3}$
$=4^6$

Solution 3) $(5^2)^5$
$=5^{2\times5}$
$=5^{10}$

Your Turn:

a) $2^2\times2^5=$

b) $3^4\div3^3=$

c) $4^7\div4^3\times4^2=$

d) $(x^5)^3=$

e) $(a^4)^3\div(a^2)^3=$

f) $\frac{c^3\times c^2}{c^7}=$

Examples: Calculate the value without a calculator. 1) 5^{-3} 2) $8^{1/3}$

Solution 1) 5^{-3}

$=\frac{1}{5^3}$

$=\frac{1}{125}$

Solution 2) $8^{1/3}$

$=\sqrt[3]{8}$

$=2$

Your Turn:

g) $3^{-2} =$

h) $7^0 =$

i) $2 \times 3^3 =$

j) $2^2 \times 3^2 =$

k) $64^{\frac{1}{2}} =$

l) $27^{\frac{1}{3}}$

3. Surds

Examples: Simplify, writing as a single surd where possible 1) $\sqrt{3} + 2\sqrt{3}$ 2) $\sqrt{2} \times \sqrt{5}$ 3) $\sqrt{90}$

Solution 1) $\sqrt{3} + 2\sqrt{3}$
$= 3\sqrt{3}$

Solution 2) $\sqrt{2} \times \sqrt{5}$
$= \sqrt{2 \times 5}$
$= \sqrt{10}$

Solution 3) $\sqrt{90}$
$= \sqrt{9 \times 10}$
$= \sqrt{9} \times \sqrt{10}$
$= 3\sqrt{10}$

Your Turn:

a) $3\sqrt{7} + 2\sqrt{7} =$

b) $4\sqrt{2} - 3\sqrt{2} =$

c) $3\sqrt{7} \times \sqrt{7} =$

d) $3\sqrt{6} \times \sqrt{6} =$

e) $\sqrt{18} =$

f) $\sqrt{32} =$

g) $\sqrt{54} =$

h) $\sqrt{12} =$

4. Expanding and Simplifying Expressions

Examples: Expand and simplify 1) $4(x + 2) - 3(x - 1)$ and 2) $(x + 4)(x - 6)$

Solution 1) $4(x + 2) - 3(x - 1)$
$= 4x + 8 - 3x + 3$
$= 4x - 3x + 8 + 3$
$= x + 11$

Solution 2) $(x + 4)(x - 6)$
$= x^2 - 24 + 4x - 6x$
$= x^2 - 2x - 24$

Your Turn:

a) $6b^2 + 5b - 1 + 3b + 4$

b) $5(x - 3)$

c) $-2(3x + 1)$

d) $3(4x + 2) + 5(2x - 1)$

e) $5(2x - 4) - 2(3x - 7)$

f) $(x - 1)(x + 1)$

g) $(3a + 2)(a - 1)$

h) $(2b - 3)(3b - 2)$

i) $4(xy)$

j) $(3x)^2$

k) $\frac{3x+6y}{3}$

l) $\frac{4}{2x+4}$

m) $\frac{2}{5x-2}$

5. Factorising

Examples: 1) $9xy + 15x$ 2) $x^2 + 3x + 2$ 3) $x^2 - 9$

Solution 1) $9xy + 15x$ — write the highest common factor, HCF, outside the brackets
$3x(\quad)$ — divide both parts of the expression by the HCF
$3x(3y + 5)$ — check your answer by multiplying through the brackets.

Solution 2) $x^2 + 3x + 2$ — Set out double brackets, writing an x in each one

$(x\quad)(x\quad)$ — Think of two factors of 2 that will add to 3.

$(x + 2)(x + 1)$

Solution 3) $x^2 - 9$ — 9 and x^2 are both square numbers; this is a DOTS question!

$(x + 3)(x - 3)$ — add and subtract the square roots in the brackets.

Your Turn:

a) $4x + 8 =$

b) $2ab + ad$

c) $8x^2 - 10x$

d) $8ab^2 - 4a^2b$

e) $x^2 + 4x + 3$

f) $x^2 + 8x + 15$

g) $x^2 + 12x - 28$

h) $x^2 - 17x + 30$

i) $x^2 - 36$

j) $2x^2 + 7x + 5$

6. Solving Linear Equations

Examples:

Solve the following equations 1) $5x + 4 = 11$ and 2) $7(x-2) = 7$

Solution 1)

$$5x + 4 = 11$$
$$5x = 11 - 4$$
$$5x = 7$$
$$x = 7 \div 5 = \tfrac{7}{5}$$

Solution 2)

$$7(x-2) = 7$$
$$7x - 14 = 7$$
$$7x = 7 + 14 = 21$$
$$x = 21 \div 7 = 3$$

Your Turn:

a) $5x + 7 = 32$

b) $2(2x - 7) = 7$

c) $4x - 5 = 2x + 7$

d) $3p + 2 = 5 - p$

e) $2 - 3(2x - 5) = 7 - x$

f) $\frac{3x-13}{7} + \frac{11-4x}{3} = 0$

g) $\frac{6}{x} + \frac{3}{2x} = \frac{5}{2}$

7. Formulae

Examples: Substitute into the following formulae to determine the missing value 1) If $x = ab - c$, find x when $a = 4$, $b = \tfrac{1}{2}$ and $c = -5$

$$x = ab - c$$
$$= 4 \times \tfrac{1}{2} - (-5) \qquad 4 \times \tfrac{1}{2} = 2 \text{ and } -(-5) \text{ is the same as } +5$$
$$= 2 + 5$$
$$= 7$$

Your Turn:

a) $x = ab + c$ Find x when $a = \frac{2}{3}$, $b = 9$ and $c = -3$

b) $x = 2a^2$ Find x when $a = \frac{3}{4}$

c) $A = 4\pi r^2$ Find r when A = 616

d) $a = b - ½c$ Find c when a = 6 and b = 10

e) $v = u + at$ Find a when v = 21.5, u = 4 and t = 7

Examples: Make x the subject of each of these formulae;

1) $a = x - ab$ 2) $xy = w$ and 3) $f = d(x+e)$

Solution 1)

$a = x - ab$ Treat ab as a single item; add ab to each side

$a + ab = x$ Swap each side to give $x =$

$x = a + ab$

Solution 2)

$xy = w$ Remember that $xy = x \times y$, you need to divide by y

$x = \frac{w}{y}$

Solution 3)

$f = d(x+e)$ Firstly multiply out the brackets

$f = dx + de$ Treating de as a single item; subtract de from each side

$f - de = dx$ Divide by d

$\frac{f-de}{d} = x$ Swap each side to give $x =$

$x = \frac{f-de}{d}$

Your Turn:

f) $3x = b$

g) $\frac{x}{5} = d$

h) $f = 4 - x$

i) $2(3x - 1) = 5y$

j) $ax = bx + c$

k) $mx = u - 2x$

l) $\sqrt{x-2} = y$

8. Solving Quadratic Equations

Examples:

Solve the following quadratic equations; 1) $x^2 - 8x + 12 = 0$ and 2) $y^2 + 13y + 40 = 0$.

Solution 1)

In the quadratic equation $x^2 - 8x + 12 = 0$, the expression can be factorised. So $(x - 6)(x - 2) = 0$

We set each factor pair equal to zero to get our two solutions.

$x - 6 = 0$ and $x - 2 = 0$

x = 6 or 2

Solution 2)
In the equation $y^2+13y+40=0$, we have $a=1$, $b=13$ and $c=40$. So

$$y=\frac{-13\pm\sqrt{13^2-4\times1\times40}}{2\times1}=\frac{-13\pm\sqrt{169-160}}{2}=\frac{-13\pm\sqrt{9}}{2}=\frac{-13\pm3}{2}=\frac{-13+3}{2}\text{ or }\frac{-13-3}{2}=-5\text{ or }-8$$

Your Turn:

a) $n^2 + 5n + 4 = 0$

b) $t^2 - 4t - 12 = 0$

c) $x^2 - 81 = 0$

d) $x^2 - 2x - 6 = 0$

e) $x^2 - 6x - 8 = 0$

f) $3x^2 + 10x - 7 = 0$

9. Simultaneous Linear Equations

Examples:
Solve the following pairs of simultaneous equations

1) $7x+2y=32$
$x+y=1$

2) $5x+2y=26$
$4x-3y=7$

Solution 1)
Double the second equation to give
$7x+2y=32$
$2x+2y=2$
Subtract the new second equation from the new first, and solve the resulting equation to find x
$5x=30$
$x=6$
Substitute into either of the original equations to find y
$x+y=1$
$\Rightarrow\ 6+y=1$
$y=-5$

Solution 2)
Multiply the first equation by 3 and the second equation by 2 to give
$15x+6y=78$
$8x-6y=14$
Add the two equations and solve
$23x=92$
$x=4$
Substitute into either of the original equations to find y
$5x+2y=26$
$\Rightarrow 20+2y=26$
$2y=6$
$y=3$

Your Turn:

a) 5x – 3y = 23
2x + 3y = 26

b) y = 2x + 1
3y + 10x = 7

c) 5x + 2y = 11
3x + 7y = -5

d) x + 2y = 4
2x + y = 5

e) 3x – 6y = 33
x – 3y = 16

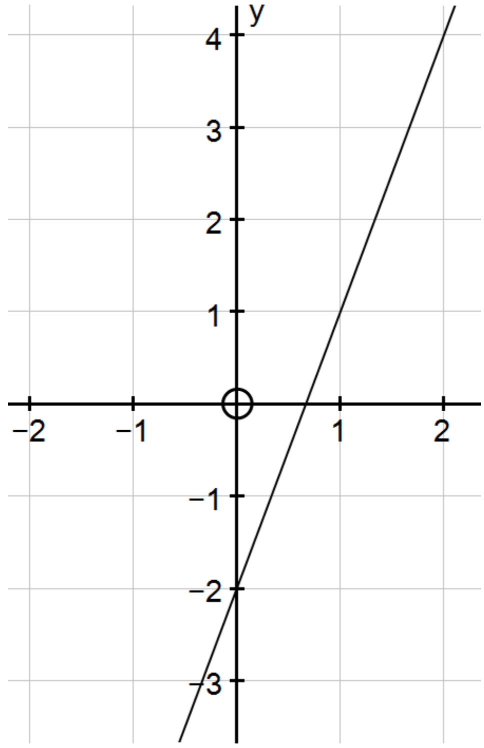
10. Straight Line Graphs

Draw the graph and state the gradient and y-intercept for each line.

Example: y = 3x – 2

Either set up a table of values to get some coordinates or go straight to the graph using the gradient and y-intercept.

x	-2	0	2
y	-8	-2	4



y = 3 x -2 – 2 = 8

y = 3 x 0 – 2 = -2

y = 3 x 2 – 2 = 4

When written in the form y = mx + c

m = gradient = $\frac{change\ in\ y}{change\ in\ x}$

c = y-intercept

for this equation, m = 3 and c = -2

Your Turn:

a) y = 2x + 1

b) $y = \frac{2}{3}x - 3$

c) x + 2y = 6

11. Completing the Square

https://www.mathsisfun.com/algebra/completing-square.html

Express in the form $(x+a)^2+b$

a x^2+2x+4	**b** x^2-2x+4	**c** x^2-4x+1	**d** x^2+6x
e x^2+4x+8	**f** x^2-8x-5	**g** $x^2+12x+30$	**h** $x^2-10x+25$
i x^2+6x-9	**j** $18-4x+x^2$	**k** x^2+3x+3	**l** x^2+x-1

Quadratic Equation can have a coefficient of a in front of x^2:

$$ax^2 + bx + c = 0$$

But that is easy to deal with ... just divide the whole equation by "a" first, then carry on:

$$x^2 + (b/a)x + c/a = 0$$

Express in the form $a(x+b)^2+c$

a $2x^2+4x+3$	**b** $2x^2-8x-7$	**c** $3-6x+3x^2$	**d** $4x^2+24x+11$
e $-x^2-2x-5$	**f** $1+10x-x^2$	**g** $2x^2+2x-1$	**h** $3x^2-9x+5$
i $3x^2-24x+48$	**j** $3x^2-15x$	**k** $70+40x+5x^2$	**l** $2x^2+5x+2$
m $4x^2+6x-7$	**n** $-2x^2+4x-1$	**o** $4-2x-3x^2$	**p** $\frac{1}{3}x^2+\frac{1}{2}x-\frac{1}{4}$

Expand out your answers to check your work.
Why do we complete the square?
Find the turning point of each of your answers above.

12. Inequalities

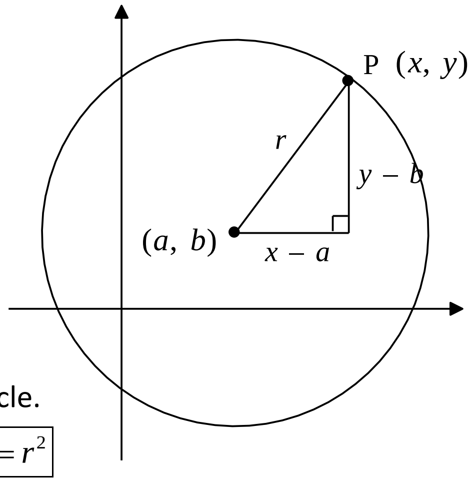
Linear inequalities are solved in the same way as linear equations. But you must keep one thing in mind: multiplying or dividing by a negative number causes the inequality to change direction. Often it is worth finding a way around dividing by the negative number in the first place to avoid mistakes.

Q1) $8x+7<47$	Q5) $4x+1>3x+3$
Q2) $4x-8<20$	Q6) $2x+54<10x+6$
Q3) $4x+3>43$	Q7) $7x+1<8x-3$
Q4) $4x-5\geq-17$	Q8) $6x+15\geq 8x-5$

13. Circles and tangents

Cartesian Equation of a Circle

Consider a circle of radius *r* and centre (*a*, *b*).



Pythagoras gives us the general Cartesian equation of the circle.

$$(x-a)^2+(y-b)^2=r^2$$

Questions:

Write down an equation of the circle with the given centre and radius in each case.

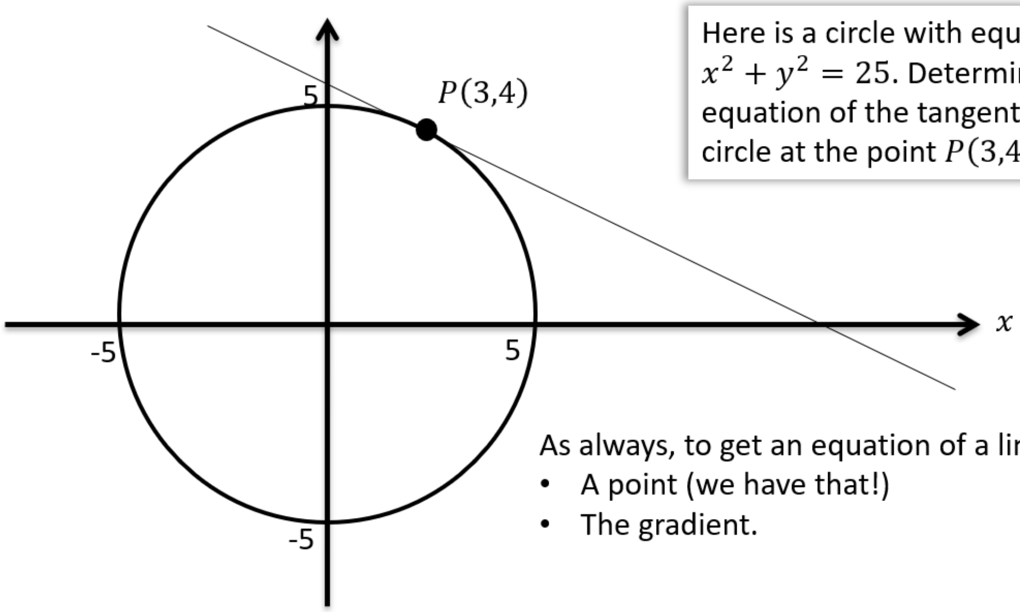
a centre (0, 0) radius 5 **b** centre (1, 3) radius 2 **c** centre (4, −6) radius 1

d centre (−1, −8) radius 3 **e** centre $(-\frac{1}{2}, \frac{1}{2})$ radius $\frac{1}{2}$ **f** centre (−3, 9) radius $2\sqrt{3}$

Write down the coordinates of the centre and the radius of each of the following circles.

a $x^2+y^2=16$ **b** $(x-6)^2+(y-1)^2=81$ **c** $(x+1)^2+(y-4)^2=121$

d $(x-7)^2+y^2=0.09$ **e** $(x+2)^2+(y+5)^2=32$ **f** $(x-8)^2+(y+9)^2=108$



Here is a circle with equation $x^2+y^2=25$. Determine the equation of the tangent of the circle at the point $P(3,4)$.

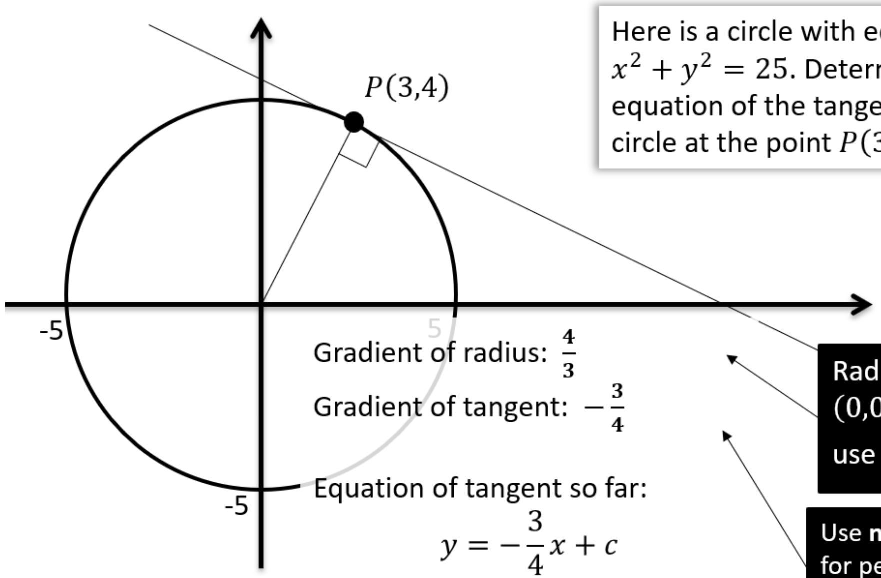
As always, to get an equation of a line we need:

- A point (we have that!)
- The gradient.

There's only ONE thing you need to remember for this topic, related to finding the gradient of the tangent:

✐ The tangent is perpendicular to the radius.

Here is a circle with equation $x^2 + y^2 = 25$. Determine the equation of the tangent of the circle at the point $P(3,4)$.



Gradient of radius: $\frac{4}{3}$

Gradient of tangent: $-\frac{3}{4}$

Equation of tangent so far:

$$y = -\frac{3}{4}x + c$$

At $\boldsymbol{P}$: $4 = -\frac{3}{4} \times 3 + c$

$$4 = -\frac{9}{4} + c$$

$$c = \frac{25}{4} \rightarrow y = -\frac{3}{4}x + \frac{25}{4}$$

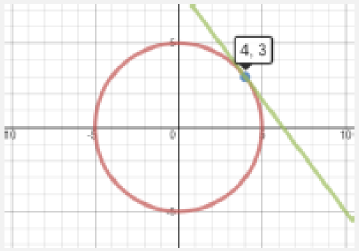
Radius goes through $(0,0)$ and $(3,4)$ so use $m = \frac{change\ in\ y}{change\ in\ x}$

Use **negative reciprocal** for perpendicular gradient.

We know $P(3,4)$ is a point on the line so substitute into equation to find c.

4. Find the equation of the tangent to $x^2 + y^2 - 20 = 0$ at $(-4, -2)$	6. Find the equation of the tangent to the circle with centre $(0, 0)$ and diameter $\sqrt{32}$ at the point $(2, 2)$
5. Find the equation of the tangent to $x^2 + y^2 = 13$ at the point on the circumference with x-coordinate 3 and a negative y-coordinate	7. Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(5, 0)$

Equations of Tangents to Circles

Question	Radius	Sketch	Gradient of radius at point	Gradient of tangent at point	Equation of tangent at point
Example: Find the equation of the tangent to $x^2 + y^2 = 25$ at $(4, 3)$	5		$\frac{3}{4}$	$-\frac{4}{3}$	$y = -\frac{4}{3}x + \frac{25}{3}$
1. Find the equation of the tangent to $x^2 + y^2 = 5$ at $(2, 1)$					
2. Find the equation of the tangent to $x^2 + y^2 = 100$ at the point on the circumference with x-coordinate 6 and a positive y-coordinate					
3. Find the equation of the tangent to $x^2 + y^2 = 45$ at $(-6, 3)$					

14. Harder simultaneous equations

More difficult simultaneous equations involve quadratics and need to be solved using **substitution**. This type of question gives 4 answers, rather than 2 in normal simultaneous equations.

Examples:

Solve the following pairs of simultaneous equations

$$x^2 + y^2 = 100$$
$$x - y = 2$$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$$x = y + 2$$

Step 2- substitute into the quadratic and simplify

$(y + 2)^2 + y^2 = 100$
$y^2 + 4y + 4 + y^2 = 100$
$2y^2 + 4y - 96 = 0$
$y^2 + 2y - 48 = 0$

Step 3- Solve by factorising or using the formula

$(y + 8)(y - 6) = 0$
$y = -8$ or $y = 6$

Step 4- Substitute your two values into the linear equation to find the other solutions

$x = y + 2$
When $y = -8$, $x = -8 + 2 =$ **-6**
When $y = 6$, $x = 6 + 2 =$ **8**

$$y = x^2 - 3x + 4$$
$$y - x = 1$$

Step 1- rearrange the linear equation if necessary. X or y needs to be the subject

$$y = 1 + x$$

Step 2- substitute into the quadratic and simplify

$1 + x = x^2 - 3x + 4$
$x^2 - 4x + 3 = 0$

Step 3- Solve by factorising or using the formula

$(x - 1)(x - 3) = 0$
$x = 1$ or $x = 3$

Step 4- Substitute your two values into the linear equation to find the other solutions

$y = 1 + x$
When $x = 1$, $y = 1 + 1 =$ **2**
When $x = 3$, $y = 1 + 3 =$ **4**

Your Turn:

1. $y = x^2 + 7x - 2$
   $y = 2x - 8$
2. $x^2 + y^2 = 8$
   $y = x + 4$
3. $y = x^2$
   $y = x + 2$
4. $x^2 + y^2 = 5$
   $x - 2y = 5$
5. $x^2 + y^2 = 36$
   $x = 2y + 6$
6. $x^2 + y^2 = 25$
   $y = 2x + 5$
7. $x^2 + y^2 = 9$
   $x + y = 2$

15. Trigonometry

Look up and learn to plot the graphs for sin, cos and tan. Bring a copy of these graphs.
You should already know the exact values of certain trigonometric ratios for your GCSE. Fill in the following table and learn the values.

θ	0°	30°	45°	60°	90°
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

You will be expected to know the following trigonometric formulae off by heart, and be able to recognise when each is relevant.

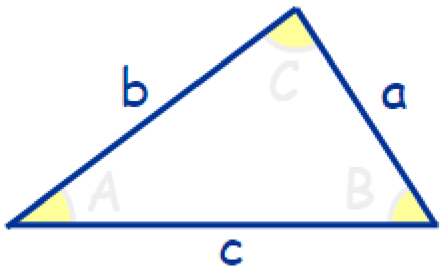
Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

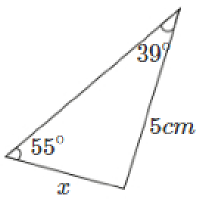
Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

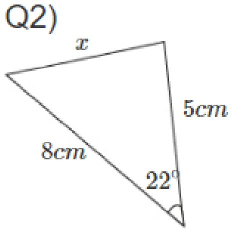
$$Area\ of\ a\ triangle = \frac{1}{2}ab\sin C$$



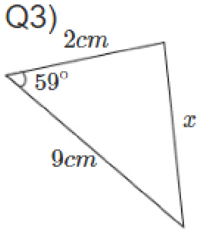
Find x $(2dp)$:
Q1)



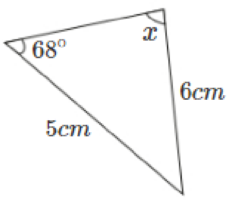
Find x $(2dp)$:
Q2)



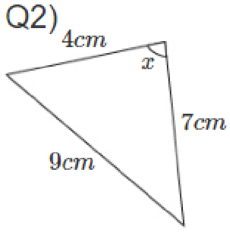
Find x $(2dp)$:
Q3)



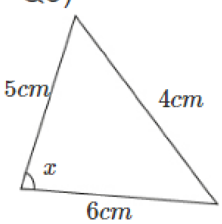
Find x to the nearest degree:
Q1)



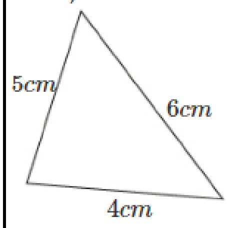
Find x to the nearest degree:
Q2)



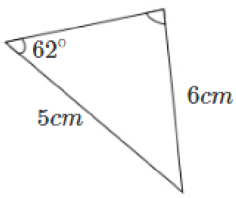
Find x to the nearest degree:
Q3)



Find the area $(2dp)$:
Q1)



Find the area $(2dp)$:
Q2)



Find the area $(2dp)$:
Q3)